第5章 互感电路

- 5.1 互感系数和耦合系数
  - **1.互感系数（coefficient of mutual inductance）**
    - **互感的性质**
      - 互感系数M只与两个线圈的几何尺寸、匝数、相对位置和周围的介质磁导率有关。
      - 自感正比于线圈的匝数平方，互感正比于两个线圈的匝数之积。
  - **2.耦合系数（coupling coefficient）**
- 5.2 互感电压及同名端
  - **1.互感电压**
  - **2.同名端（Dot Convention）**
- 5.3 互感元件的连接和去耦等效电路
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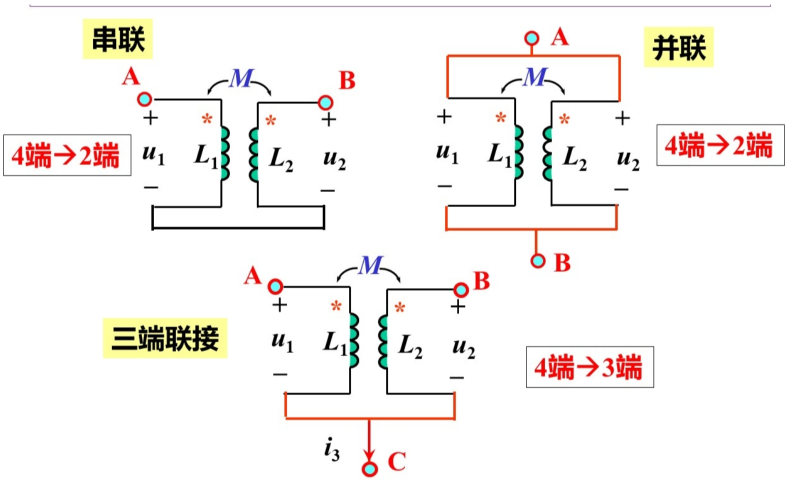
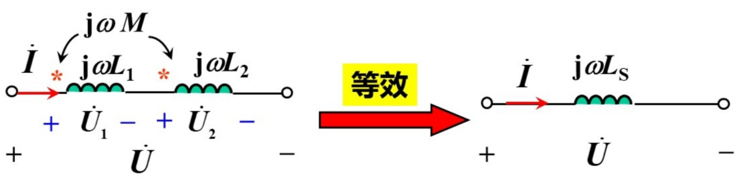


  - **1.互感串联**
    - **1）顺联**



由KVL和互感的伏安关系，顺联时可以得到：

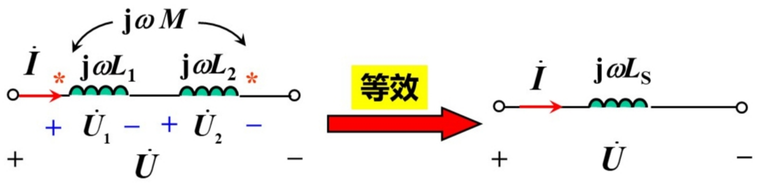
$$\dot{U}=\dot{U}_1+\dot{U}_2=\left(\mathrm{j}\omega L_1\dot{I}+\mathrm{j}\omega M\dot{I}\right)+\left(\mathrm{j}\omega L_2\dot{I}+\mathrm{j}\omega M\dot{I}\right)$$

$$=\mathrm{j}\omega\left(L_1+L_2+2M\right)\dot{I}$$

$$=\mathrm{j}\omega L_S\dot{I}$$

$$L_S=L_1+L_2+2M$$

    - **2）逆联**



由KVL和互感的伏安关系，逆联时可以得到：

$$\dot{U}=\dot{U}_1+\dot{U}_2=\left(\mathrm{j}\omega L_1\dot{I}-\mathrm{j}\omega M\dot{I}\right)+\left(\mathrm{j}\omega L_2\dot{I}-\mathrm{j}\omega M\dot{I}\right)$$
$$=\mathrm{j}\omega\left(L_1+L_2-2M\right)\dot{I}$$
$$=\mathrm{j}\omega L_S\dot{I}$$
$$L_S=L_1+L_2-2M$$

- 问题：如何测量互感值?

$$L_{顺}=L_1+L_2+2M \qquad L_{逆}=L_1+L_2-2M$$

* **顺联一次，逆联一次，就可以测出互感：**

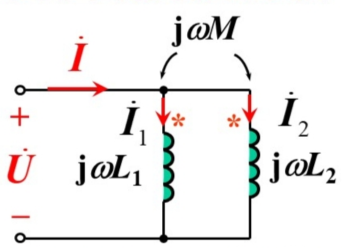
$$M=\frac{L_{顺}-L_{逆}}{4}$$

* **全耦合** $M=\sqrt{L_1L_2}$

当 $L_1=L_2=L$ **时，** $M=L$

$$L_S=\begin{cases}4M & \text{顺联}\\ 0 & \text{逆联}\end{cases}$$

- 2.互感并联
  - 1）同名端同侧并联



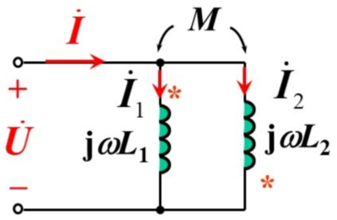
$$\begin{cases}\dot{U}=\mathrm{j}\omega L_1\dot{I}_1+\mathrm{j}\omega M\dot{I}_2\\ \quad=\mathrm{j}\omega L_2\dot{I}_2+\mathrm{j}\omega M\dot{I}_1\\ \dot{I}=\dot{I}_1+\dot{I}_2\end{cases}$$

解得电压与电流的关系

$$\dot{U}=\mathrm{j}\omega\frac{(L_1L_2-M^2)}{L_1+L_2-2M}\dot{I}$$

$$L_p=\frac{(L_1L_2-M^2)}{L_1+L_2-2M}$$

  - 2）同名端异侧并联



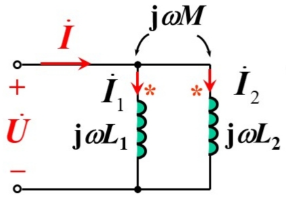
$$\begin{cases} \dot{U} = \mathrm{j}\omega L_1\dot{I}_1 - \mathrm{j}\omega M\dot{I}_2 \\ \quad = \mathrm{j}\omega L_2\dot{I}_2 - \mathrm{j}\omega M\dot{I}_1 \\ \dot{I} = \dot{I}_1 + \dot{I}_2 \end{cases}$$

解得电压与电流的关系

$$\dot{U} = \mathrm{j}\omega\frac{(L_1L_2 - M^2)}{L_1 + L_2 + 2M}\dot{I}$$

$$L_p = \frac{(L_1L_2 - M^2)}{L_1 + L_2 + 2M} \quad \geq 0$$

- 同名端在同侧并联电路的去耦等效分析



$$\begin{cases} \dot{U} = \mathrm{j}\omega L_1\dot{I}_1 + \mathrm{j}\omega M\dot{I}_2 \\ \dot{U} = \mathrm{j}\omega L_2\dot{I}_2 + \mathrm{j}\omega M\dot{I}_1 \end{cases}$$

$\dot{I}_2 = \dot{I} - \dot{I}_1$

$\dot{I}_1 = \dot{I} - \dot{I}_2$

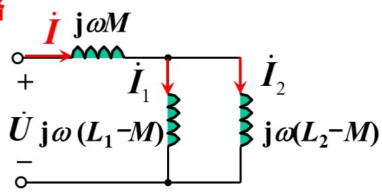
$$\begin{cases} \dot{U} = \mathrm{j}\omega(L_1 - M)\dot{I}_1 + \mathrm{j}\omega M\dot{I} \\ \dot{U} = \mathrm{j}\omega(L_2 - M)\dot{I}_2 + \mathrm{j}\omega M\dot{I} \end{cases}$$

公共部分

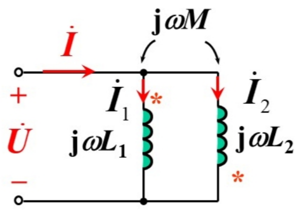
画等效电路

$(L_1-M)//(L_2-M)+M$

$$L_p = \frac{(L_1L_2 - M^2)}{L_1 + L_2 - 2M}$$



- 同名端在异侧并联电路的去耦等效分析

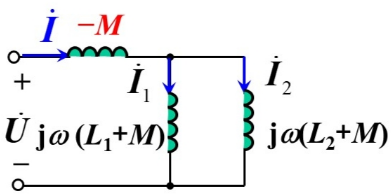


$$\begin{cases} \dot{U} = \mathrm{j}\omega(L_1 + M)\dot{I}_1 - \mathrm{j}\omega M\dot{I} \\ \dot{U} = \mathrm{j}\omega(L_2 + M)\dot{I}_2 - \mathrm{j}\omega M\dot{I} \end{cases}$$

等效电路

$(L_1+M)//(L_2+M) - M$

$$L_p = \frac{(L_1L_2 - M^2)}{L_1 + L_2 + 2M}$$



- 3.有一个公共节点同侧相联互感线圈的去耦等效
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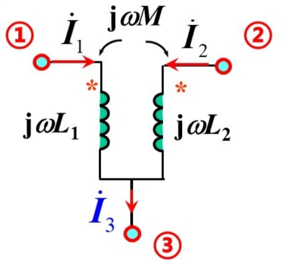


2个同名端都靠近（远离）公共节点

$$\dot{I}_3 = \dot{I}_1 + \dot{I}_2$$

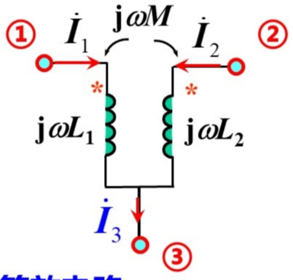
$$\dot{U}_{13} = \mathrm{j}\omega L_1 \dot{I}_1 + \mathrm{j}\omega M \dot{I}_2$$

$$= \mathrm{j}\omega (L_1 - M)\dot{I}_1 + \mathrm{j}\omega M \dot{I}_3$$

$$\dot{I}_3 = \dot{I}_1 + \dot{I}_2$$

$$\dot{U}_{23} = \mathrm{j}\omega L_2 \dot{I}_2 + \mathrm{j}\omega M \dot{I}_1$$

$$= \mathrm{j}\omega (L_2 - M)\dot{I}_2 + \mathrm{j}\omega M \dot{I}_3$$

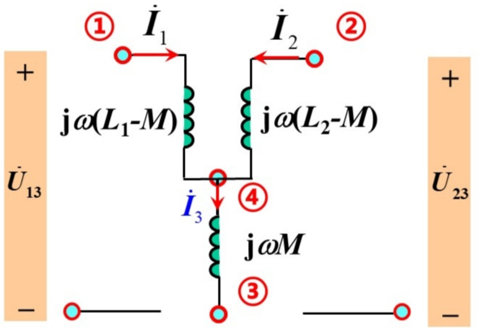


$$\dot{U}_{13} = \mathrm{j}\omega (L_1 - M)\dot{I}_1 + \mathrm{j}\omega M \dot{I}_3$$

$$\dot{U}_{23} = \mathrm{j}\omega (L_2 - M)\dot{I}_2 + \mathrm{j}\omega M \dot{I}_3$$

$$\dot{I}_3 = \dot{I}_1 + \dot{I}_2$$

等效电路

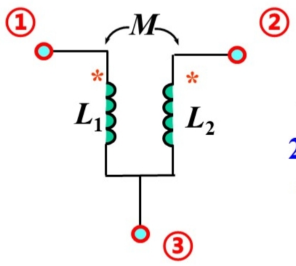


强调：

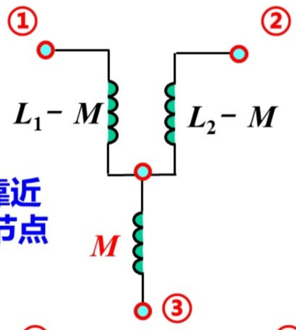
多了个节点④

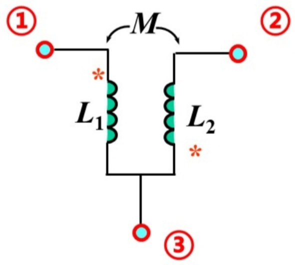
$\dot{U}_{13} \neq \dot{U}_{14}$

$\dot{U}_{23} \neq \dot{U}_{24}$

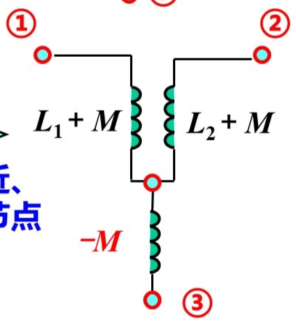


2个同名端都靠近（远离）公共节点





同名端1个靠近、1个远离公共节点

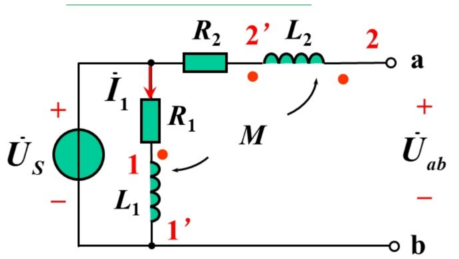


- 注意

1、去耦等效后，不必再考虑互感作用和互感电压，互感作用已体现在各等效电感中。但应用时注意对外等效，即对互感元件以外电路是无影响的，互感元件内部结构已变，无对应关系。

2、去耦结果只与互感元件连接结构及同名端位置决定，与电压电流参考方向无关。

- 5.4 具有互感的正弦电路的分析
  - 实际互感元件同名端的判别方法
    - **1.交流判别法**

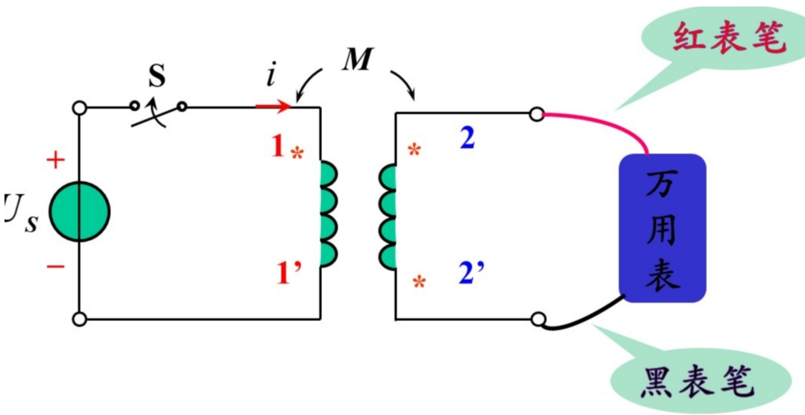


如图所示连接两个线圈，利用信号源输出正弦电压 $\dot{U}_S$ 作为激励，利用毫伏表分别测量$_S$ 和$_{ab}$ 。

若 $U_{ab} > U_S$，则1与2为同名端

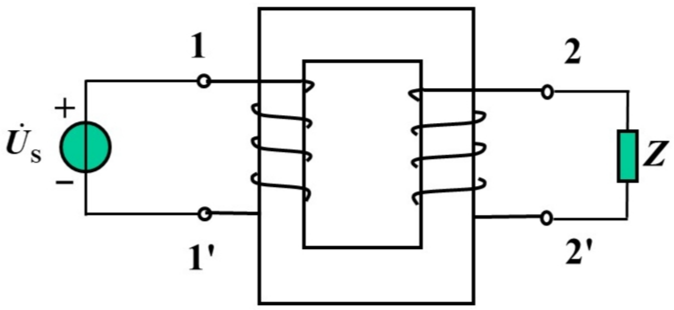
若 $U_{ab} < U_S$，则1与2'为同名端

    - **2.直流判别法**



当开关闭合，线圈通电瞬间，若万用表指针正偏或所测电压为正值，则1和2为同名端。
若反偏或示数为负值，则1和2’为同名端。

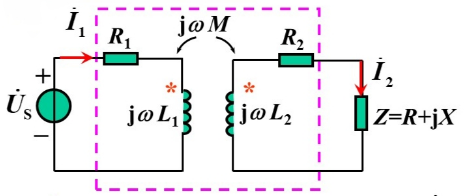
- 5.5 变压器原理
  - 变压器（Transformer）



利用互感的作用来传递能量

- **阻抗变换**
- **电隔离**
- **传送功率**
- **交流变压、变流**

- 1.空心变压器



原边回路总抗阻

$Z_{11}=R_1+j\omega L_1$

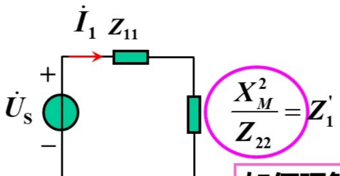
副边回路总阻抗

$Z_{22}=(R_2+R)+j(\omega L_2+X)$

$$\begin{cases} Z_{11}\dot{I}_1 - j\omega M\dot{I}_2 = \dot{U}_S \\ -j\omega M\dot{I}_1 + Z_{22}\dot{I}_2 = 0 \end{cases} \qquad Z_{in} = \frac{\dot{U}_S}{\dot{I}_1} = Z_{11} + \frac{(\omega M)^2}{Z_{22}} = Z_{11} + \frac{X_M^2}{Z_{22}}$$

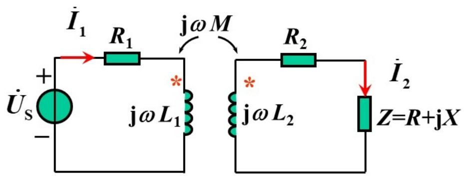
$$\dot{I}_2 = \frac{j\omega M\dot{I}_1}{Z_{22}}$$

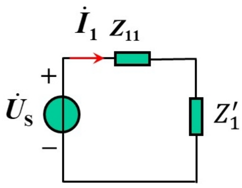
$$\dot{I}_1 = \frac{\dot{U}_S}{Z_{11} + \dfrac{(\omega M)^2}{Z_{22}}}$$



原边等效电路

如何理解?





原边等效电路

$Z_1' = R_1' + jX_1'$ **副边反映在原边回路中的阻抗（引入阻抗）。**

$$Z_{in} = \frac{\dot{U}_S}{\dot{I}_1} = Z_{11} + \frac{(\omega M)^2}{Z_{22}}$$

引入电阻 **引入电抗**

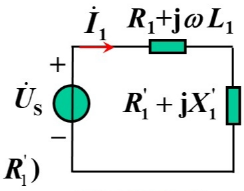
$$Z_1' \overset{\text{def}}{=} \frac{(\omega M)^2}{Z_{22}} = \frac{\omega^2 M^2}{R_{22} + jX_{22}} = \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} - j\frac{\omega^2 M^2 X_{22}}{R_{22}^2 + X_{22}^2} = R_1' + jX_1'$$

当 $\dot{I}_2 = 0$, **即副边开路** $Z_{in} = Z_{11}$

当 $\dot{I}_2 \neq 0$, $Z_{in} = Z_{11} + Z_1'$ **变压器实现电抗性质变化**

能量角度分析

$$R_1' = \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} > 0$$



原边看：

电源发出有功＝电阻吸收有功＝$I_1^2(R_1+R_1')$

$I_1^2R_1$ **消耗在原边；** $\boxed{I_1^2 R_1'?}$

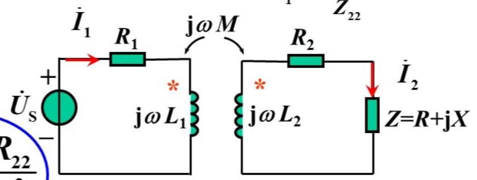
$Z_{11}=R_1+\mathrm{j}\omega L_1$

$Z_{22}=(R_2+R)+\mathrm{j}(\omega L_2+X)$

$Z_1' \overset{\text{def}}{=} \frac{(\omega M)^2}{Z_{22}}$

副边看：

$$\dot{I}_2 = \frac{\mathrm{j}\omega M \dot{I}_1}{Z_{22}}$$

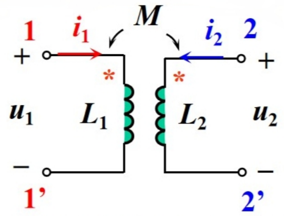


副边吸收的有功：

$$I_2^2 R_{22} = I_1^2 \times \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} = I_1^2 R_1'$$

变压器实现功率传送

- 2.理想变压器（ideal transformer）

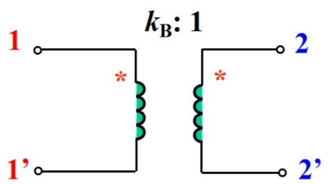


$$\psi_1 = N_1 \Phi$$
$$\psi_2 = N_2 \Phi$$

$$u_1 = \frac{\mathrm{d}\psi_1}{\mathrm{d}t} = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}i_2}{\mathrm{d}t}$$
$$u_2 = \frac{\mathrm{d}\psi_2}{\mathrm{d}t} = L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} + M\frac{\mathrm{d}i_1}{\mathrm{d}t}$$

在全耦合情况下，两个线圈的磁通Φ相同，所以有

$$\frac{u_1}{u_2} = \frac{\dfrac{\mathrm{d}\psi_1}{\mathrm{d}t}}{\dfrac{\mathrm{d}\psi_2}{\mathrm{d}t}} = \frac{\dfrac{\mathrm{d}N_1\Phi}{\mathrm{d}t}}{\dfrac{\mathrm{d}N_2\Phi}{\mathrm{d}t}} = \frac{N_1}{N_2} = k_B$$



功率角度分析 $u_1 i_1 = -u_2 i_2$

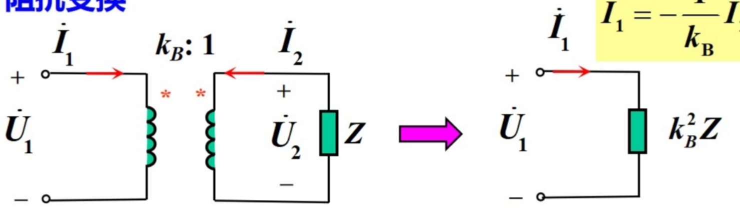
电流比为 $\frac{i_1}{i_2} = -\frac{u_2}{u_1} = -\frac{1}{k_B}$

理想变压器的电路模型

理想变压器的性质：

$$\dot{U}_1 = k_B \dot{U}_2$$
$$\dot{I}_1 = -\frac{1}{k_B}\dot{I}_2$$

(a) 阻抗变换



$$Z_{in} = \frac{\dot{U}_1}{\dot{I}_1} = \frac{k_B \dot{U}_2}{-\dfrac{1}{k_B}\dot{I}_2} = -k_B^2 \frac{\dot{U}_2}{\dot{I}_2} = -k_B^2 Z$$

(b) 功率消耗

$$p = u_1 i_1 + u_2 i_2 = k_B u_2 \times (-\frac{1}{k_B}) i_2 + u_2 \times i_2 = 0$$

理想变压器既不储能，也不耗能，
在电路中只起传递信号和能量的作用。